Chapter 1. Newton's Laws of Motion

Notes:

- *Most of the material in this chapter is taken from Young and Freedman, Chapters 4 and 5*

1.1 Forces and Interactions

It was Isaac Newton who first introduced the concepts of **mass** and **force**, to a large extent to make sense of the experimental results obtained by previous scientists. Using these concepts, or principles, he was able to put forth three fundamental laws of motions (i.e., **Newton's Laws of Motion**) upon which much of classical physics rests upon[1]. We review the different types of forces encountered in **Newtonian** (or **classical**) **mechanics** before we introduce Newton's Laws.

A force is an interaction between two bodies or between a body and its environment. One intuitive type of force is a **contact force**, which often clearly involves a direct interaction (or contact) between the surfaces or boundaries of the bodies involved. We can further discriminate between different kinds of contact forces.

1. A body in contact with the surface of another object will experience a force that is directed normal to that surface. Perhaps the simplest example of a **normal force** is that of a book resting on a table. Since the book does not fall under its own weight it must be that the table is exerting a force normal to its surface to keep the book at rest. Another example is that of a block at rest or sliding on an inclined plane. Even in the case where the block is sliding, there is the presence of a force oriented perpendicularly to the surface of the plane; if not the block would not be sliding but falling through the plane.
2. The aforementioned normal force is not the only one exerted by the plane on an object sliding on it. There is a **friction force** that is oriented parallel to the surface of contact, but in the direction opposite to that of the sliding. Friction forces can be further differentiated depending on whether the object is not sliding (but is about to) or it is already in motion. More precisely, it takes a greater applied force to start the object moving (i.e., to overcome the **static friction force**) than it takes to keep it sliding (when the **kinetic friction force** is at work).
3. Finally, whenever an object is pulled, through a string or a rope attached to it, it can be set in motion through a **tension force** (as long as it is stronger than the static friction force).

[1] Exceptions to this include anything involving elementary particles or atoms and molecules (i.e., the realm of quantum mechanics) as well as classical electromagnetism. The inconsistency of Newton's Laws with Maxwell's equations of electromagnetism led to the development of special relativity, by A. Einstein, which expands upon (and corrects) Newton's Laws to situations involving motions nearing the speed of light.

Contact forces are not the only agents through which bodies interact, however. For example, electric and magnetic forces as well as the gravitational force act over distances in vacuum and therefore do not require any type of contact (or a series of *contacts*) to make themselves felt by bodies. Such forces are referred to as **long-range forces**.

1.1.1 The Principle of Superposition

Evidently more than one force or type of force can applied at once to an object. One can imagine that our block sliding on an inclined plane through the influence of gravity could at the same time be slowed downed through the use of a rope pulled by someone standing above the block on the plane. Such a block would then be subjected to four different forces: a normal force, a friction force, a tension force, and a long-range (gravitation) force. The question arises then as to how these forces combine when acting on a body. As it turns out their combined effect is additive[2]. That is, we can apply the **principle of superposition** and add all the forces vectorially (since forces are vectors, i.e., they have a magnitude and a direction).

Mathematically this is expressed as follows, given a set of n forces $\mathbf{F}_i$ for $i = 1, 2, \ldots, n$ acting on an object the **resultant** or **net force** $\mathbf{R}$ felt by this object is

$$\mathbf{R} = \sum_{i=1}^{n} \mathbf{F}_i . \tag{1.1}$$

When expressed using Cartesian coordinates the forces can be broken with their components along the x-, y-, and z-axis such that

$$\begin{aligned} R_x &= \sum_{i=1}^{n} F_{x,i} \\ R_y &= \sum_{i=1}^{n} F_{y,i} \\ R_z &= \sum_{i=1}^{n} F_{z,i} . \end{aligned} \tag{1.2}$$

For example, for the case of our previous sliding block we have

$$\mathbf{R} = \mathbf{N} + \mathbf{f} + \mathbf{T} + \mathbf{G}, \tag{1.3}$$

where $\mathbf{N}, \mathbf{f}, \mathbf{T}, \text{and } \mathbf{G}$ are the normal, friction, tension, and gravitational forces, respectively.

[2] Here we assume that the body can be appropriately modeled as a single point or that all the forces are applied at its centre of mass.

1.2 Newton's Laws

Now that we have determined the kind of forces that are susceptible to affect the dynamics of bodies in Newtonian mechanics, we now turn to the laws of nature that will allow us to quantify these interactions. Newton's Laws are often simply stated as:

I. *A body remains at rest or in uniform motion unless acted upon by a net force.*

II. *A body acted upon by a net force moves in such a manner that the time rate of change of the momentum equals the force.*

III. *If two bodies exert forces on each other, these forces are equal in magnitude and opposite in direction.*

The First Law would be meaningless without the concept of force, but it conveys a precise meaning for the concept of a "zero net force". This tendency for a body to remain in its initial state of motion (or at rest) is called **inertia**. One should note that according to the first law, there is no way to distinguish between "no net force" and "no force at all". That is, the only thing that matter is the resultant. It is irrelevant whether two or three forces (or any number for that matter) are applied simultaneously, if they cancel each other out, then their effect (or lack thereof) is the same as that of having no force at all applied to the body; the body will remain in its initial state of uniform motion. We then say that the body is in **equilibrium** (since it is not influenced by forces).

The Second Law is very explicit: Force is the time rate of change of the **momentum**. But what is the momentum $\mathbf{p}$ …

$$\mathbf{p} \equiv m\mathbf{v}, \tag{1.4}$$

with m the mass, and $\mathbf{v}$ the velocity of the body. We therefore rewrite the Second Law as

$$\begin{aligned} \mathbf{F}_{\text{net}} &= \frac{d\mathbf{p}}{dt} \\ &= \frac{d}{dt}(m\mathbf{v}). \end{aligned} \tag{1.5}$$

Although we still don't have a definition for the concept of mass, we can further transform equation (1.5), if we assume that it is a constant, to yield

$$\begin{aligned} \mathbf{F}_{\text{net}} &= m\frac{d\mathbf{v}}{dt} \\ &= m\mathbf{a}, \end{aligned} \tag{1.6}$$

with $\mathbf{a} \equiv d\mathbf{v}/dt$ the acceleration resulting from the action of the net force on the body. Note that the acceleration is in the same direction as the force and proportional to it (the constant of proportionality being the mass).

The concept of mass is made clear with the Third Law, which can be rewritten as follows:

III'. *If two bodies constitute an ideal, isolated system, then the accelerations of these bodies are always in opposite direction, and the ratio of the magnitudes of the accelerations is constant. This constant ratio is the inverse ratio of the masses of the bodies.*

If we have two isolated bodies, 1 and 2, then the Third Law states that

$$\mathbf{F}_1 = -\mathbf{F}_2, \tag{1.7}$$

and from the Second Law we have

$$\frac{d\mathbf{p}_1}{dt} = -\frac{d\mathbf{p}_2}{dt}, \tag{1.8}$$

or using the acceleration **a**

$$\begin{aligned} m_1\mathbf{a}_1 &= -m_2\mathbf{a}_2 \\ \frac{m_1}{m_2} &= \frac{a_2}{a_1}, \end{aligned} \tag{1.9}$$

with $a_i = |\mathbf{a}_i|$. If one chooses m_1 as the reference or unit mass, m_2, or the mass of any other object, can be measured by comparison (if it is allowed to interact with m_1) of their measured accelerations. Incidentally, we can use equation (1.8) to provide a different interpretation of Newton's Second Law

$$\frac{d}{dt}(\mathbf{p}_1 + \mathbf{p}_2) = 0 \tag{1.10}$$

or

$$\mathbf{p}_1 + \mathbf{p}_2 = constant. \tag{1.11}$$

The momentum is conserved in the interaction of two isolated particles. This is a special case of the **conservation of linear momentum**, which is a concept that we will discuss at length later on.

One should note that the Third Law is not a general law of nature. It applies when dealing with **central forces** (e.g., gravitation (in the non-relativistic limit), electrostatic, etc.), but not necessarily to other types of forces (e.g., velocity-dependent forces such as between to moving electric charges). But such considerations are outside the scope of our studies.

1.2.1 Inertial Frames of Reference

The concept of an **inertial frame of reference** is central to the application of Newton's Laws. An inertial frame is one that is in a uniform and non-accelerating state of motion. In fact, Newton's Laws are only applicable to such frame of references. Conversely, Newton's First Law can be used to define what an inertial frame is.

For example, suppose that a body that is not subjected to any net force is in a dynamical state that is in accordance with the First Law, as seen by an observer at rest in a given frame of reference. One would therefore define this frame as being inertial (for that reason Newton's First Law is often call the **law of inertia**). Now if a second observer at rest in another frame sees that the body is not moving in a uniform motion, then this second frame of reference cannot be inertial. This may due, for example, to the fact that this frame of reference is itself accelerating in some fashion, which would account for the apparent non-uniform motion of the observed body. It is often stated that Newton's Laws are **invariant** when moving from one inertial frame to another. This can be better understood mathematically.

If a particle of mass m has a velocity $\mathbf{u}'$ relative to an observer who is at rest an inertial frame K', while this frame is moving with at a constant velocity $\mathbf{v}$ as seen by another observer at rest in another inertial frame K (note the necessity of having a constant relative velocity $\mathbf{v}$), then we would expect that the velocity $\mathbf{u}$ of the particle as measured in K to be

$$\mathbf{u} = \mathbf{u}' + \mathbf{v}. \tag{1.12}$$

That is, it would seem reasonable to expect that velocities should be added together when transforming from one inertial frame to another[3] (such a transformation is called a **Galilean transformation**). If we write the mathematical form of the Second Law in frame K we have

$$\begin{aligned} \mathbf{F} &= m\frac{d\mathbf{u}}{dt} \\ &= m\frac{d\left(\mathbf{u}' + \mathbf{v}\right)}{dt}, \end{aligned} \tag{1.13}$$

but since $\mathbf{v}$ is constant

$$\mathbf{F} = m\frac{d\mathbf{u}'}{dt} = \mathbf{F}'. \tag{1.14}$$

[3] The development of special relativity has showed that this law for the composition of velocities is approximate and only valid when these are small compared to the speed of light.

The result expressed through equation (1.14) is a statement of the invariance (or covariance) of Newton's Second Law. More precisely, the Second Law retains the same mathematical form no matter which inertial frame is used to express it, as long as velocities transform according to the simple addition rule stated in equation (1.12).

1.2.2 Exercises

1. (Prob. 4.8 in Young and Freedman.) You walk into an elevator, step onto a scale, and push the "up" button. You also recall that your normal weight is 625 N. Start answering each the following questions by drawing a free-body diagram.

a) If the elevator has an acceleration of magnitude of $2.50\ \text{m/s}^2$, what does the scale read?
b) If you start holding a 3.85-kg package by a light vertical string, what will be the tension in the string once the elevator begins accelerating?

Solution.

a) The elevator and everything in it are accelerating upward, so we apply Newton's Second Law in the vertical direction only. Your mass is determined with $m = w/g = 625\ \text{N}/9.8\ \text{N kg}^{-1} = 63.8\ \text{kg}$, but you and the package have the same acceleration as the elevator. Taking $+y$ as the upward direction we use the free-body diagram of Figure 1a), where n is the scale reading, we calculate

$$\begin{aligned}\sum F_y &= n - w \\ &= ma\end{aligned} \tag{1.15}$$

or

$$\begin{aligned}n &= w + ma \\ &= 625\ \text{N} + 63.8\ \text{kg} \cdot 2.50\ \text{m/s}^2 \\ &= 784\ \text{N}.\end{aligned} \tag{1.16}$$

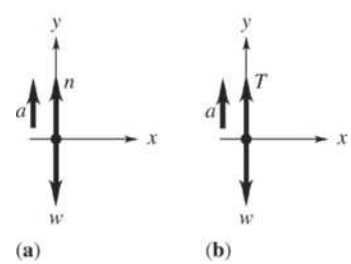


Figure 1 – Free-body diagram.

b) Referring to the free-body diagram of Figure 1b) we write

$$\begin{aligned}\sum F_y &= T - w_\text{p} \\ &= m_\text{p} a\end{aligned} \tag{1.17}$$

or

$$\begin{aligned}T &= w_\text{p} + m_\text{p} a \\ &= 3.85\ \text{kg} \cdot \left(9.80\ \text{m/s}^2 + 2.50\ \text{m/s}^2\right) \\ &= 47.4\ \text{N}.\end{aligned} \tag{1.18}$$

2. (Prob. 4.57 in Young and Freedman.) Two boxes, *A* and *B*, are connected to each end of a light rope (see Figure 2). A constant upward force of 80.0 N is applied to box *A*. Starting from rest, box *B* descends 12.0 m in 4.00 s. The tension in the rope connecting the two boxes is 36.0 N. What are the masses of the two boxes?

Solution.

The system is accelerating, so we apply Newton's second law to each box and can use the constant acceleration kinematics for formulas to find the acceleration. We now that

$$\mathbf{a} = \frac{d^2\mathbf{x}}{dt^2} \tag{1.19}$$

and therefore

$$\mathbf{x}(t) = \int_{-\infty}^{t} \left[\int_{-\infty}^{\tau} \mathbf{a}(\lambda)\, d\lambda \right] d\tau \tag{1.20}$$

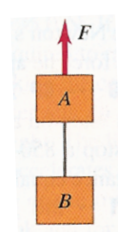


Figure 2

In one dimension (along the y-axis) when the acceleration is constant equation (1.20) becomes

$$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2, \tag{1.21}$$

where y_0 and v_{0y} are, respectively, the position and velocity at $t = 0$. We therefore have for box B ($v_{0y} = 0$)

$$\begin{aligned} a_y &= \frac{2(y - y_0)}{t^2} \\ &= \frac{2 \cdot (-12.0 \text{ m})}{16 \text{ s}^2} \\ &= -1.5 \text{ m/s}^2. \end{aligned} \tag{1.22}$$

Alternatively we can write (defining $g > 0$)

$$m_B a_y = T - m_B g, \tag{1.23}$$

or

$$\begin{aligned} m_B &= \frac{T}{g + a_y} \\ &= \frac{36.0 \text{ N}}{(9.8 - 1.5)\text{m/s}^2} \\ &= 4.34 \text{ kg}. \end{aligned} \tag{1.24}$$

And for box A

$$m_A a_y = F - T - m_A g, \tag{1.25}$$

or

$$\begin{aligned} m_A &= \frac{F - T}{g + a_y} \\ &= \frac{(80.0 - 36.0) \text{ N}}{(9.8 - 1.5)\text{m/s}^2} \\ &= 5.30 \text{ kg}. \end{aligned} \tag{1.26}$$

Note that even though the boxes have the same acceleration they experience different forces because they have different masses.

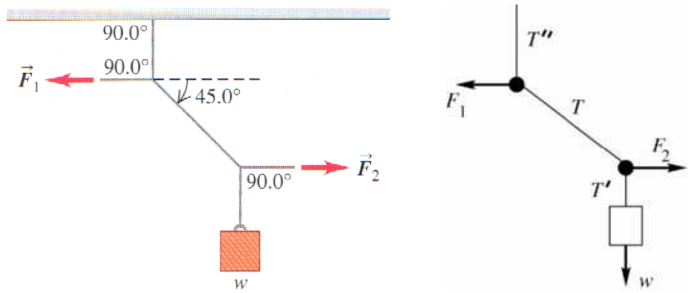


Figure 3 – System for Problem 3 (left) and free-body diagram (right).

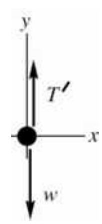
3. (Prob. 5.10 in Young and Freedman.) In Figure 3 the weight $w = 60.0$ N .

a) What is the tension in the diagonal string?
b) Find the magnitudes of the horizontal forces $\mathbf{F}_1$ and $\mathbf{F}_2$ that must be applied to hold the system in the position shown.

Solution.

Newton's first law will suffice for this problem as the system is in equilibrium. We can apply it to the hanging weight and to each knot. The tension force at each end of a string is the same.

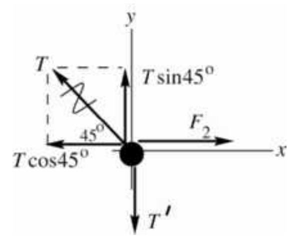
a) Let the tensions in the three strings be T, T', and T'', as shown in Figure 3. The free-body diagram for the block is



We can therefore write

$$\begin{aligned}\sum F_y &= T' - w \\ &= 0,\end{aligned} \tag{1.27}$$

or $T' = w = 60.0$ N . The free-body diagram for the lower knot is



And we have

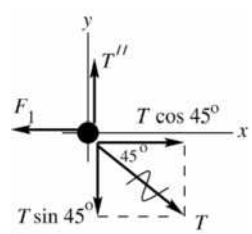
$$\sum F_y = T\sin\left(45^\circ\right) - T' \\ = 0, \tag{1.28}$$

which yields $T = \sqrt{2}T' = 84.9\ \text{N}$.

b) We now apply Newton's first law in the x direction. For the lower knot we have from the above figure

$$\sum F_x = T\cos\left(45^\circ\right) - F_2 \\ = 0, \tag{1.29}$$

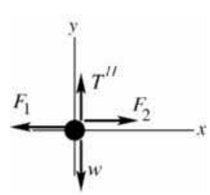
or $F_2 = T/\sqrt{2} = 60.0\ \text{N}$. For the upper knot we have the following free-body diagram



With a similar calculation we find that $F_1 = F_2 = 60.0\ \text{N}$. Finally, we also see that

$$T'' = T\sin\left(45^\circ\right) \\ = 60.0\ \text{N}. \tag{1.30}$$

These last two results could have been easily predicted by treating the whole system as a single object and studying the corresponding force diagram



1.3 Friction Forces Revisited

We have already briefly discussed friction forces. We now seek to quantify them in relation to another contact force, i.e., the normal force. The nature of the friction force is very complicated. At its basis, it is a microscopic phenomenon involving the making and breaking of molecular bonds between the two contacting surfaces. It should therefore not be expected that any friction force is constant over time as an object is sliding on a surface (obviously we are referring here to the kinetic friction force). This is because the number of bonds created or destroyed will vary depending on the roughness of the surfaces at different position or the inhomogeneous presence of alien matter between them (e.g., dirt or oil). It follows that our assigning of a single, constant force for a given problem is an approximation, i.e., it must represent some sort of macroscopic average that results from the detailed microphysical phenomena that take place.

1.3.1 Kinetic Friction

It is found experimentally that the kinetic friction force is proportional to the normal force of contact. If $\mathbf{f}_k$ and $\mathbf{n}$ represent these two forces, then they can be related through

$$|\mathbf{f}_k| = \mu_k |\mathbf{n}|, \tag{1.31}$$

where μ_k is the **coefficient of kinetic friction**. The norms ($|\cdots|$) in equation (1.31) are not used and this relation will commonly be written as

$$f_k = \mu_k n. \tag{1.32}$$

It should be noted that neither of equations (1.31) or (1.32) are vectorial in nature, since $\mathbf{f}_k$ and $\mathbf{n}$ are perpendicular to one another.

1.3.2 Static Friction

As was stated earlier, friction does not imply motion. In fact, it is usually the case that friction is stronger when an objet is immobile. More precisely, the static friction force $\mathbf{f}_s$ on a body is normally stronger than the kinetic friction that settles in once it starts moving. We know from experiments that the maximum magnitude that the static friction force can take, $(f_s)_{\max}$, is proportional to the normal contact force. It follows that

$$f_s \leq \mu_s n, \tag{1.33}$$

where μ_s is the **coefficient of static friction**. Experimental measurements show that typical values range (approximately) from $0.05 < \mu_s < 1$ (but it can be greater than unity), while $\mu_k < \mu_s$ in general.

1.3.3 Rolling Friction

Interestingly, although the less friction there is between a body on a surface the better the former slides on the latter, the same is not true for rolling motions. That is, a good rolling action depends on the presence of some friction; rolling would not take place in the limit where there is no friction. On the other hand, **rolling friction** and its corresponding **coefficient or rolling friction** μ_r, defined as *the ratio of the force needed for constant speed to the normal force exerted by the surface over which the rolling motion takes place*, is significantly lower than the coefficient of kinetic friction μ_k. Typical values for μ_r range from 10^{-3} to 10^{-2}. If you have to move a heavy object, you may do well to put wheels under it …

1.3.4 Fluid Resistance and Friction

The definition of a fluid can encompass a whole variety of agent. A fluid could be loosely defined as a substance (gas or liquid) that deforms under shear stress and easily yields to external pressure. Some examples include the atmosphere (air), oils, or the rarefied agglomerations of matter (i.e., gases made of molecules and dust) in the interstellar medium.

If we take the case of an object in motion in the atmosphere, one could model this (solid) body-fluid interaction by considering the many collisions involving the molecules that make the atmosphere on the surface of the body. These interactions will transfer linear momentum between the colliding partners and affect the motion and kinetics of the body as it progresses on its path through the fluid. We have yet to study collisions in detail but the motion of an object within a fluid can nonetheless be determined experimentally.

For objects moving a low speed relative to the surrounding fluid the friction, or **drag force** resulting from the numerous body-fluid collisions is found to be proportional to the speed v

$$f = kv, \tag{1.34}$$

with k the proportionality constant. This friction force is directed in opposition to the direction of motion. For larger objects moving at higher speeds (on the order of, say, 10 m/s) the drag force is proportional to the square of the velocity

$$f = Dv^2. \tag{1.35}$$

Newton's Second Law can be used to gain more insight, so let us consider an object of mass m sinking into a fluid under the effect of gravity. We therefore find

$$mg - f = ma. \tag{1.36}$$

For motions occurring at low speed where equation (1.34) applies the solutions to equation (1.36) for the velocity $v(=dy/dt)$ and the position y as a function of time can be readily shown to be

$$\begin{aligned} v(t) &= \frac{mg}{k}\left(1-e^{-kt/m}\right) \\ y(t) &= \frac{mg}{k}\left[t-\frac{m}{k}\left(1-e^{-kt/m}\right)\right], \end{aligned} \tag{1.37}$$

respectively, for $t>0$. The acceleration is given by (also when $t>0$)

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= ge^{-kt/m}. \end{aligned} \tag{1.38}$$

We find that the velocity, which we assume to be $v=0$ at $t=0$, eventually reaches a maximum, **terminal velocity** when $t=\infty$ (when $a=0$, obviously)

$$v_{\mathrm{t}} = \frac{mg}{k}. \tag{1.39}$$

Since this terminal velocity is constant it could have easily been determined with equation (1.36) when equilibrium is reached (i.e., when $a=0$). We would then have that

$$mg - kv_{\mathrm{t}} = 0, \tag{1.40}$$

from which we recover equation (1.39). The non-linear nature of the friction force at higher speed (i.e., equation (1.35)) renders it impossible to calculate solutions similar to those presented in equations (1.37). But we can still consider equilibrium conditions to determine a terminal velocity

$$v_{\mathrm{t}} = \sqrt{\frac{mg}{D}}. \tag{1.41}$$

The existence of a terminal velocity is entirely due to the fact that the friction force scales with the velocity of the object. This is to be contrasted with the case of contact friction forces, which are independent of the velocity.

1.4 Circular Motions

Consider an object that is initially moving with a uniform, rectilinear motion at velocity v. One might ask what force must be applied to the object such that it enters into a circular orbit of radius R at the same speed. Referring to Figure 4 we see that over an

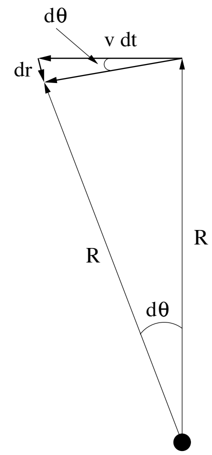


Figure 4 – Motion of a body over an infinitesimal amount of time t . To go from a rectilinear to a circular motion of radius R , its linear path must be modified by an amount dr .

infinitesimal time dt the object must "fall" a (radial) distance dr towards the centre of the circular orbit.

Again referring to the figure, we can write

$$dr = \left(v\,dt\right)d\theta \tag{1.42}$$

and

$$v\,dt = R\,d\theta. \tag{1.43}$$

We will now define two quantities: the angular velocity

$$\begin{aligned}\omega &\equiv \frac{d\theta}{dt}\\ &= \frac{v}{R}\end{aligned} \tag{1.44}$$

and the radial acceleration that the body must have in order to get into the circular orbit

$$\begin{aligned}a_{\text{rad}} &\equiv \frac{dv_{\text{rad}}}{dt}\\ &= \frac{1}{dt}\left(\frac{dr}{dt}\right)\\ &= v\frac{d\theta}{dt}\end{aligned} \tag{1.45}$$

which when using equation (1.44) becomes

$$
\begin{aligned}
a_{\text{rad}} &= \frac{v^2}{R} \\
&= \omega^2 R.
\end{aligned} \tag{1.46}
$$

This radial acceleration is more commonly known as the **centripetal acceleration**. Because an object moving in uniform motion will remain as such unless subjected to a non-zero net force (Newton's First Law), circular motion (with a constant orbital velocity) will only be realized if a net force oriented radially toward the centre of the orbit act on the object to induce a centripetal acceleration as defined in equation (1.46). This net force must enter Newton's Second Law when dealing with problems involving circular motions.

1.4.1 Exercises

4. (Prob. 5.42 in Young and Freedman) A small car with a mass of 0.8 kg travels at constant speed on the inside of a track that is a vertical circle with a radius of 5.0 m. If the normal force exerted by the track on the car when it is at the top of the track is 6.0 N, what is the normal force on the car when it is at the bottom of the track? What is the speed of the car?

Solution.

Two forces are acting on the car, gravity and the normal force. At the top, both forces are toward the center of the circle, so Newton's second law gives

$$mg + n_B = ma_{\text{rad}}. \tag{1.47}$$

At the bottom, gravity is downward but the normal force is upward, so

$$n_A - mg = ma_{\text{rad}}. \tag{1.48}$$

Solving equation (1.47) for the acceleration

$$
\begin{aligned}
a_{\text{rad}} &= g + \frac{n_B}{m} \\
&= 9.8 \text{ m/s} + \frac{6\,\text{N}}{0.8 \text{ kg}} \\
&= 17.3 \text{ m/s}^2.
\end{aligned} \tag{1.49}
$$

We can now determine that

$$
\begin{aligned}
n_A &= m\left(g + a_{\text{rad}}\right) \\
&= 0.8 \text{ kg}\left(9.8 + 17.3\right) \text{ m/s}^2 \\
&= 21.7 \text{ N.}
\end{aligned} \tag{1.50}
$$

The normal force at the bottom is greater than at the top because it must balance the weight in addition to accelerate the car toward the center of its track. Finally, the speed of the car is found to be (with equation (1.46)

$$
\begin{aligned}
v &= \sqrt{a_{\text{rad}} R} \\
&= \sqrt{17.3 \text{ m/s}^2 \cdot 5 \text{ m}} \\
&= 9.3 \text{ m/s.}
\end{aligned} \tag{1.51}
$$